ELECTROMAGNETIC DISTURBANCE CREATED BY AN EXPANDING CONDUCTING CYLINDER IN A MAGNETIC FIELD

V. K. Bodulinskii and Yu. A. Medvedev

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 4, pp. 91–92, 1969

We examine the electromagnetic disturbance created by a linearly expanding, ideally conducting cylinder in an external uniform magnetic field with account for effects of order v/c (v is the boundary velocity).

The problem of uniform magnetic field compression in a contracting cylindrical cavity with ideally conducting walls with radius decreasing linearly with time was solved in [1]. As far as we know the corresponding external problem has not been investigated.

In the problem of the expansion of an ideally conducting cylinder with linearly increasing radius $a(t) = vt$ in an external constant uniform magnetic field parallel to the cylinder axis, the components $H_z(r, t) \equiv H$ and $E_\varphi(r, t) \equiv -E$ are nonzero and satisfy the equations

$$\frac{\partial H}{\partial r} = \frac{1}{c}\frac{\partial E}{\partial t} \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE) = \frac{1}{c}\frac{\partial H}{\partial t} \tag{2}$$

(we have used the r, z cylindrical coordinate system with z-axis along the cylinder axis) and the initial conditions

$$H(r, 0) = H_0, \qquad E(r, 0) = 0 \tag{3}$$

The following boundary condition must be satisfied on the moving surface of the cylinder [2]:

$$E(a(t), t) + \frac{a^{\cdot}(t)}{c} H(a(t), t) = 0 \tag{4}$$

We seek the solution of the problem (1)–(4) in the region $\{a(t) \le r < \infty, t \ge 0\}$, regular at infinity and satisfying the radiation principle.

The solutions must have the form of an expanding wave; therefore, it is convenient to convert to the variables ρ and τ:

$$\rho = r, \qquad \tau = t - r / c \tag{5}$$

in which Eqs. (1) and (2) take the form

$$\frac{\partial H}{\partial \rho} - \frac{1}{c}\frac{\partial H}{\partial \tau} = \frac{1}{c}\frac{\partial E}{\partial \tau}, \qquad \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E) - \frac{1}{c}\frac{\partial E}{\partial \tau} = \frac{1}{c}\frac{\partial H}{\partial \tau} \tag{6}$$

We can formulate the problem for only the function H or for only E. Then in the problem of the magnetic field H there are three independent similarity criteria:

$$H / H_0, \ \beta = v / c, \ v\tau / \rho$$

and on the basis of the pi-theorem [3] we have the relations

$$\frac{H}{H_0} = f_1\left(\beta, \frac{v\tau}{\rho}\right), \qquad \frac{E}{H_0} = f_2\left(\beta, \frac{v\tau}{\rho}\right)$$

In the case in question β = const; then we seek the functions H and E in the form

$$H = H_0 \Phi(x), \qquad E = H_0 \Psi(x), \qquad x = v\tau / \rho \tag{7}$$

Substituting (7) into (6), we obtain

$$(x+\beta)\,\Phi' = -\beta\Psi', \qquad \Psi - (x+\beta)\,\Psi' = \beta\Phi' \tag{8}$$

(the prime denotes differentiation with respect to x), having the solutions

$$\Psi(x) = A\sqrt{x^2+2\beta x} \qquad (A = \mathrm{const})$$

$$\Phi(x) = -\beta A\ln\left[2\sqrt{x^2+2\beta x} + 2(x+\beta)\right] + B \qquad (B = \mathrm{const})$$

Substituting the last relations into (7), returning to the original variables r and t, and determining the constants A and B from the initial (3) and boundary (4) conditions, we obtain

for $vt \le r \le ct$

$$E(r,t) = -H_0F(\beta)\left[\left(\frac{vt-\beta r}{r}\right)^2 + 2\beta\left(\frac{vt-\beta r}{r}\right)\right]^{1/2}$$

$$H(r,t) = H_0\left\{1-\beta F(\beta)\ln\left[2\left(\frac{vt-\beta r}{r}\right)^2 + 2\beta\left(\frac{vt-\beta r}{r}\right)\right]^{1/2} + 2\left(\frac{vt-\beta r}{r}\right) + 2\beta\right]\right\}$$

$$F(\beta) = \left\{\sqrt{1-\beta^2} - \beta\ln\left[2\left(1+\sqrt{1-\beta^2}\right)\right]\right\}^{-1}$$

for $r > ct$

$$H = H_0,\ E = 0$$

We can see by direct substitution that these formulas yield the solution of the problem (1)–(4).

These results may be useful in evaluating the diamagnetic disturbances accompanying the expansion of a lightning discharge column, the decay of wakes of bodies traveling at high speed in the atmosphere in the Earth's magnetic field, and also in other processes of an explosive nature with cylindrical symmetry in an external magnetic field.

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17 June 1968

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